International Workshop on
“Fundamental Problems in Mathematical and Theoretical Physics”

Date: July 23 – July 27, 2024
Venue: 02 Conference Room, 1st Floor, 55N Bldg., Waseda University, Nishi-Waseda Campus
早稲田大学 西早稲田キャンパス 55 号館 N 棟 1 階 第 2 会議室

Abstract of Minicourses

Quantum Physics

♢ Paolo FACCHI ♢

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Product Formulas in Quantum Dynamics

- ◆ Minicourse I July 23, Tuesday 10:30 – 12:00
- ◆ Minicourse II July 23, Tuesday 16:30 – 18:00

Product formulas provide a valuable method for decomposing the dynamics of a quantum system into simpler components. These formulas possess a rich mathematical structure, and their history is extensive, with significant contributions from mathematicians and physicists such as Lie, Trotter, Feynman, Kac, Chernoff, Kato, and Nelson. Recently, these formulas have proven invaluable in quantum technologies, ranging from digital quantum simulation to quantum control and dynamical decoupling. In these lectures, I will provide an introduction to the subject, featuring several examples and recent results.

♢ Saverio PASCAZIO ♢

University of Bari, Italy

Quantum Correlations

- ◆ Minicourse I July 23, Tuesday 14:45 – 16:15
- ◆ Minicourse II July 24, Wednesday 10:30 – 12:00

Quantum correlations are fascinating and complex phenomena described by quantum physics that cannot be explained by classical physics. These correlations have intrigued scientists for decades, as they reveal the profoundly non-intuitive nature of the quantum world. The concept of quantum correlations was first explored by Albert Einstein, Boris Podolsky, and Nathan Rosen in their famous 1935 paper, which introduced the idea of quantum entanglement. This pioneering work was further developed by Erwin Schrödinger, who coined the term “entanglement” (Verschränkung in German) to describe these quantum correlations.

Thirty years after the initial explorations by Einstein, Podolsky, Rosen, and Schrödinger, John Bell made a significant breakthrough in understanding quantum correlations. In 1964, Bell formulated what is now known as Bell’s Theorem, which provided a way to test the predictions of quantum mechanics against those of classical physics. Bell’s work demonstrated that no local hidden variable theory could reproduce all the predictions of quantum mechanics, thereby solidifying the idea that quantum correlations are fundamentally different from classical correlations.

Our journey into the realm of quantum correlations begins with a historical timeline of key discoveries and milestones. We examine the initial controversies and debates that surrounded the nature of entanglement and quantum correlations, which were hotly debated topics among physicists. These early discussions laid the groundwork for the profound insights and experimental confirmations that followed.

Moving beyond the historical perspective, we delve into the physical and mathematical aspects of quantum correlations. We explore the intricate mathematical formalism that describes these correlations, such as the use of Hilbert spaces, operators, and entanglement measures.

Finally, we summarize the current status of what has been learned and is known about quantum correlations.

♢ Vladimir GEORGIEV ♢

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On ground states and well-posedness for local and nonlocal dispersive equations

- ◆ Minicourse I July 24, Wednesday 13:00 – 14:30
- ◆ Minicourse II July 25, Thursday 13:30 – 15:00
- ◆ Minicourse III July 26, Friday 10:00 – 11:30

Lecture 1: Local uniqueness of ground states for generalized Choquard problem

We consider the generalized Choquard equation of the type

$$-\Delta Q + Q = I(|Q|^p)|Q|^{p-2}Q,\ n \geq 3$$

with $Q \in H^1_{rad}(\mathbb{R}^n)$, where the operator I is the classical Riesz potential defined by $I(f)(x) = (-\Delta)^{-1}f(x)$ and the exponent $p \in (2, 1 + 4/(n-2))$ is energy subcritical. We consider Weinstein type functional restricted to rays passing through the ground state. The corresponding real valued function of the path parameter has an appropriate analytic extension. We use the properties of this analytic extension in order to show local uniqueness of ground state solutions. The uniqueness of the such ground state solutions for the case $p = 2$, i.e. for the case of Hartree–Choquard, is well known. The main difficulty for the case $p > 2$ is connected with possible lack of control on the L^p norm of the ground states as well on the lack of Sturm comparison argument.
The work is done in collaboration with George Venkov and Mirko Tarulli.

Lecture 2: On Sobolev spaces associated with Laplace operator with contact perturbation

We study the perturbed Sobolev space $H^{s,r}_\alpha$, $r \in (1, \infty)$, associated with singular perturbation Δ_α of Laplace operator in Euclidean space of dimensions $2, 3$. The main results give the possibility to extend the L^2 theory of perturbed Sobolev space to the L^r case. We have appropriate representation of the functions in $H^{s,r}_\alpha$ in regular and singular part. An application to local well - posedness of the NLS associated with this singular perturbation in the mass critical and mass supercritical cases is established too.
The talk is based on collaboration with Alessandro Michelangeli, Mario Rastrelli and Raffaele Scandone.

Lecture 3: On damped wave equation in dimension 1: global solution and decay in Fujita subcritical case.

We consider the Cauchy problem:

$$\begin{cases} \partial_t^2 u + \partial_t u - \Delta u = |u|^p, & (t,x) \in (0,\infty) \times \mathbb{R}^n, \\ u(0,x) = u_0(x), & x \in \mathbb{R}^n, \\ \partial_t u(0,x) = u_1(x), & x \in \mathbb{R}^n. \end{cases} \tag{1}$$

The aim is to estimate the decay rate of u with $n = 1$ in the L^∞ framework under the pointwise initial condition

$$u_0 \leq 0, \quad u_1 \leq -u_0/2. \tag{2}$$

We remark that the local well-posedness of (1) is known.

The main result is the following. Let $u_0 \in C^2 \cap W^{2,1} \cap W^{2,\infty}$ and $u_1 \in C^1 \cap L^1 \cap L^\infty$ satisfy (2). Then the corresponding solution u to (1) satisfies

$$\sup_{t \geq 0} (1+t)^{1/(p-1)} \|u(t)\|_{L^\infty} \leq \infty$$

if $p > 5/3$.

The talk is based on collaboration with Kazumasa Fujiwara.

♢ Kenji NAKANISHI ♢

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Global wellposedness of general evolution equations on the Fourier half space

◆ Minicourse I	July 24, Wednesday	15:00 – 16:30
◆ Minicourse II	July 25, Thursday	10:00 – 11:30
◆ Minicourse III	July 26, Friday	13:30 – 15:00

This minicourse will explain from scratch my recent joint work (arXiv:2401.09746) with Baoxiang Wang (Jimei/Peking), where we obtain unique global solutions of the initial data problem for extraordinarily wide classes of initial data and nonlinear partial differential equations, under the restriction that the Fourier transform of solution is supported in the half space.

The class of equations covers not only physically important ones from fluid mechanics, nonlinear waves and nonlinear diffusion, but also pathological ones that are illposed in the classical settings. The class of initial data contains both decaying functions and periodic ones, as well as many others, and beyond the space of tempered distributions. Although our function space of the solution is a bit weird, our main result is the global wellposedness in the classical Hadamard sense.

A major drawback of the Fourier support restriction is that the solutions cannot be non-trivial real valued, yet they contain some interesting cases such as stationary solutions of the KdV equation with algebraic decay, and (complex) blow-up of the viscous Burgers and the Navier-Stokes equations.

I will also show by examples how we have the global solutions when they blow up in the usual sense, and how they differ from other extensions beyond singularities.