International Workshop on
"Fundamental Problems in Mathematical and Theoretical Physics"

Date: July 22 – July 26, 2025
Venue: 02 Conference Room, 1st Floor, 55N Bldg., Waseda University, Nishi-Waseda Campus
早稲田大学 西早稲田キャンパス 55 号館 N 棟 1 階 第 2 会議室

Abstract of Minicourses

Quantum Physics

♢ Paolo Facchi ♢

University of Bari, Italy

From Micro to Macro, There and Back Again

◆ Minicourse I	July 22, Tuesday	10:30 – 12:00
◆ Minicourse II	July 23, Wednesday	10:30 – 12:00
◆ Minicourse III	July 24, Thursday	10:30 – 12:00

We explore the interplay between the microscopic unitary evolution of a system-plus-environment and the effective macroscopic (non-unitary) dynamics of the system. We first derive the quantum master equation by a reduction procedure, starting with the unitary dynamics of a system coupled to its environment. Through a systematic limit approach, we arrive at the celebrated Gorini-Kossakowski-Lindblad-Sudarshan form that governs the effective evolution of open quantum systems. We then reverse the direction: given a master equation or a quantum channel, we show how it can be embedded into a unitary evolution on an extended Hilbert space. Using the Stinespring dilation theorem and the Kraus-Sudarshan representation, we uncover the hidden unitarity behind apparent irreversibility. This journey from micro to macro and back again reveals the structure of open quantum system dynamics and the deep link between decoherence, dissipation and entanglement in quantum theory.

♢ Saverio Pascazio ♢

University of Bari, Italy

Can decay be attributed to classical noise?

◆ Minicourse I	July 22, Tuesday	14:45 – 16:15
◆ Minicourse II	July 23, Wednesday	14:45 – 16:15

The dynamics of a dissipative quantum system, in the Markov approximation, is governed by the Gorini-Kossakowski-Lindblad-Sudarshan (GKLS) equation or "master" equation. Quantum dissipation can take different forms and is associated with different physical scenarios. Among them, there are true "dephasing" processes, as well as true "decay" processes (e.g., to the ground state). Consequently, the GKLS equations have different mathematical features and physical meaning. We ask ourselves the following questions here: are these different physical and mathematical features reflected in deeper properties? More specifically: can the decay be attributed to "classical" noise? This will lead us to discuss quantum Langevin equations and stochastic calculus, both in the Ito and Stratonovich formulations.

♢ Daniel Burgarth ♢

University of Erlangen-Nürnberg, Germany

The Wrong Quantum Physics: Counterexamples to Simple Proofs

◆ Minicourse I July 22, Tuesday 16:30 – 18:00
◆ Minicourse II July 23, Wednesday 16:30 – 18:00

A particle in the box with a Hamiltonian that cannot be represented by its matrix elements in a given basis? A harmonic oscillator which does not have equally spaced energy levels? An orbital angular momentum with a spectrum that contains non-integer multiples of Planck's constant? In this lecture I give counterexamples to simple proofs found in textbooks and research papers. They reveal tacit assumptions — and sometimes big gaps — in these proofs. Often, these counterexamples clearly show the "wrong" physics. Or do they?

Mathematical Physics

♢ Vladimir Georgiev ♢

Department of Mathematics, University of Pisa, Italy and
Faculty of Science and Engineering, Waseda University, Tokyo, Japan

Qualitative properties of minimizers of Hamiltonians with nonlocal interactions

◆ Minicourse I July 24, Thursday 13:30 – 14:30
◆ Minicourse II July 25, Friday 11:10 – 12:10
◆ Minicourse III July 26, Saturday 10:00 – 11:00

Lecture 1: Local uniqueness of ground states for generalized Choquard problem ($p > 2$)
Abstract: We consider the generalized Choquard equation of the type

$$-\Delta Q + Q = I(|Q|^p)|Q|^{p-2}Q, \ n \geq 3 \tag{1}$$

with $Q \in H^1_{rad}(\mathbb{R}^n)$, where the operator I is the classical Riesz potential defined by $I(f)(x) = (-\Delta)^{-1}f(x)$ and the exponent $p \in (2, 1+4/(n-2))$ is energy subcritical. We consider Weinstein type functional restricted to rays passing through the ground state. The corresponding real valued function of the path parameter has an appropriate analytic extension. We use the properties of this analytic extension in order to show local uniqueness of ground state solutions. The uniqueness of the such ground state solutions for the case $p = 2$, i.e. for the case of Hartree–Choquard, is well known. The main difficulty for the case $p > 2$ is connected with possible lack of control on the L^p norm of the ground states as well on the lack of Sturm comparison argument.

Lecture 2: Uniqueness, nondegeneracy of ground states for generalized Choquard problem ($p < 2$)
Abstract: We continue the study of (1) in the case $p < 2$.

The first goal is to obtain the uniqueness. Our goal is to explain the following result.

Assume $n \geq 3$ and $(n+2)/n < p < 2$. Then for any two positive minimizers $Q_1, Q_2 \in H^1$ of the Weinstein functional, satisfying the Pohozhaev normalization conditions we have $Q_1 = Q_2$.

To treat the nondegeneracy property we linearize around the unique ground state Q and then we study the kernel of the corresponding operator L_+.

Lecture 3: Coercive estimates. Further discussion and some open problems
Abstract: The goal is to discuss the following assertion. Assume

$$1 + \frac{2}{n} < p < 2.$$

Then we have

$$\langle L_+ h, h\rangle_{L^2} \geq C\|h\|^2_{H^1},\ \forall\, h \in \mathcal{H}_Q, \tag{2}$$

where

$$\mathcal{H}_Q = \left\{ h \in H^1,\ h \perp L_+(Q),\ h \perp \mathrm{Ker} L_+ \right\}. \tag{3}$$

♢ Nicola Visciglia ♢

Department of Mathematics, University of Pisa, Italy

Global H^2 solutions for the generalized DNLS on $\mathbb{T}$

◆ Minicourse I	July 24, Thursday	15:00 – 16:00
◆ Minicourse II	July 25, Friday	10:00 – 11:00
◆ Minicourse III	July 26, Saturday	11:10 – 12:10

Lecture 1: The main aim aim of the minicourse will be the analysis of the Cauchy problem associated with the generalized DNLS (gDNLS):

$$\begin{cases} i\partial_t u + \partial_x^2 u + i|u|^{2\sigma}\partial_x u = 0, & (t,x) \in \mathbb{R}\times\mathbb{T}, \quad \sigma \geq 1 \\ u(0,x) = \varphi \end{cases}$$

for large times. Along the first lecture we present some basic energy estimates that, combined with a compactness argument, allow to deduce the existence and uniqueness of a local solution for initial data in H^2.

Lecture 2: In the second lecture we show a global existence result, indeed we show that the H^2 norm of the solutions cannot grow faster that exponentially, provided that the H^1 norm of the solution is uniformly bounded. The main tool will be the introduction of a stuibale energy that allows to apply a Gronwall type argument. In particular we get the existence and uniqueness of a global solution by assuming the smallness of the initial datum in H^1.

Lecture 3: In the third and last lecture we use the dispersion in conjunction with energy estimates, in order to show that the exponential upper bound of the H^2 solution can be updated to a polynomial one.