International Workshop on
“Fundamental Problems in Mathematical and Theoretical Physics”

Date: July 16 – July 22, 2026
Venue: 01 Conference Room (July 16 only), 02 Conference Room, 1st Floor, 55N Bldg., Waseda University, Nishi-Waseda Campus
早稲田大学 西早稲田キャンパス 55 号館 N 棟 1 階 第 1 会議室 （7/16 のみ）, 第 2 会議室

Abstract of Minicourses

Mathematical Physics

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On completeness of modified wave operators for defocusing NLS

- ◆ Minicourse I July 16, Thursday 14:00 – 15:30
- ◆ Minicourse II July 17, Friday 14:00 – 15:30
- ◆ Minicourse III July 18, Saturday 11:40 – 12:40

The mini-course is based on results obtained in collaboration with Tohru Ozawa.

Lecture 1: Strong Limit of Modified Dynamics for Scattering-Critical NLS in Dimensions 1 and 2.

Abstract: We address the following question: given initial data u_0 in an appropriate weighted Sobolev space, what is the leading term in the asymptotic behavior of the solution $u(t)$ as $t \to \pm\infty$? More precisely, we seek a final state u^+ as $t \to +\infty$ (in a space similar to that of u_0) such that

$$\left\| u(t) - e^{i\Phi_t(u)} U(t) u^+ \right\|_{L^2} = 0 \quad \text{as } t \to +\infty. \tag{1}$$

This result provides the modified profile of the leading term, where the linear propagator $U(t)$ for the linear Schrödinger equation is perturbed by a phase correction $\Phi_t u$.

Lecture 2: Existence of modified asymptotic profile with and without decay assumption.

Abstract: We discuss uniform a priori bounds for the solution to the defocusing NLS. On the basis of these a priori bounds, we study the decay of the solution as time tends to ∞. This analysis clarifies the role of the decay assumption in the strong limit, which measures the L^2-norm of the difference between the solution to the NLS and its modified profile. Weak limits of the form

$$\lim_{t \to +\infty} |U(-t) \exp\left(i\Phi_t(u)\right) u(t) - u_+| \, \mathcal{F}(\dot{B}^{-1}2, \infty) = 0, \tag{2}$$

are discussed in dimensions 1 and 2.

Lecture 3: Strong L^q Limit of Modified Dynamics for Scattering-Critical NLS in Dimension 2.

Abstract: We discuss modified scattering for the two-dimensional defocusing nonlinear Schrödinger equation with a gauge – invariant, scattering-critical nonlinearity of order 2. Our goal is to prove that the modified wave operator, defined by

$$\lim_{t \to \pm\infty} U(-t) e^{-i\Phi_t(u)} u(t), \tag{3}$$

where $u(t)$ is a solution to the nonlinear Schrödinger equation, Φ_t is an appropriate phase function and $U(t)$ is the free propagator, is well-defined in L^2_{loc} as well as in every space L^q with $2 < q < \infty$.

This extends Yajima's results on the existence of wave operators in L^q spaces for linear Schrödinger Hamiltonians with short – range potentials.

♢ Luigi Forcella ♢

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On scattering for mass-subcritical non-scaling invariant NLS

◆ Minicourse I	July 16, Thursday	16:00 – 17:30
◆ Minicourse II	July 17, Friday	10:30 – 12:00
◆ Minicourse III	July 18, Saturday	10:30 – 11:30

The mini-course is based on results obtained in collaboration with Jacopo Bellazzini and Vladimir Georgiev.

Lecture 1: Introduction: the problem and review of classical results. Goal of the mini-course.

Abstract: We will consider the following Cauchy problem:

$$\begin{cases} i\partial_t\psi + \Delta_x\psi = |\psi|^{q-1}\psi - |\psi|^{p-1}\psi \\ \psi(0) = \psi_0 \in \Sigma_x \end{cases}, \tag{1}$$

where $\psi = \psi(t, x)$, $\psi : \mathbb{R} \times \mathbb{R}^d \mapsto \mathbb{C}$, Δ_x is the standard Laplace operator with respect to the space variables x, and $1+\frac{2}{d} < q < p < 1+\frac{4}{d}$, and the initial datum ψ_0 belongs to the weighted Sobolev space $\Sigma = \mathcal{F}H^1 = \{f \in H^1 : f \in L^2(|x|^2dx)\}$. Problem (1) is globally well-posed, i.e., a unique solution exists in $\psi \in C_t(\mathbb{R}, \Sigma_x)$. Moreover, after the pseudo conformal transformations $\tau := \frac{t}{1+t}$, $\xi := \frac{x}{1+t}$,

$$\varphi(\tau, \xi) := \psi\left(\frac{\tau}{1-\tau}, \frac{\xi}{1-\tau}\right)(1-\tau)^{-d/2}e^{-\frac{i|\xi|^2}{4(1-\tau)}},$$

we have that φ solves

$$i\partial_\tau\varphi + \Delta_\xi\varphi = (1-\tau)^{-\delta(q)}|\varphi(\tau)|^{q-1}\varphi(\tau) - (1-\tau)^{-\delta(p)}|\varphi(\tau)|^{p-1}\varphi(\tau). \tag{2}$$

We aim to prove suitable growth estimates on the kinetic and potential energy of φ, which will imply scattering in the L^2-topology. Specifically, we will prove that under a small condition on the mass of the initial datum ψ_0, $e^{\frac{i|x|^2}{4}}U(-\tau)\varphi(\tau)$ admits a strong limit in L^2 for $\tau \to 1^-$, the latter being equivalent to the scattering of ψ, namely to the existence of a state $\psi^+ = \psi^+(x)$ such that

$$\lim_{t\to+\infty} \|U(-t)\psi(t) - \psi^+\|_{L^2} = 0. \tag{3}$$

Lecture 2: Variational problems.

Abstract: An obstacle to the validity of the scattering property (3) are the so-called standing waves, i.e., solutions to (1) of the form $\psi(t, x) = e^{i\omega t}u(x)$. Consequently, $u(x)$ solves $-\Delta u + \omega u + |u|^{q-1}u - |u|^{p-1}u = 0$. We will find solutions of this form by looking at the following minimization problem:

$$\mathrm{I}_{\rho^2} = \inf_{S_\rho=\{u\in H^1\,:\,\|u\|_{L^2}=\rho\}} E(u), \tag{4}$$

where E is the energy functional associated to (1).
We will show the existence of a threshold mass $\rho_0 > 0$ such that the infimum I_ρ in (4) is negative and

achieved if $\rho > \rho_0$, while $I_\rho = 0$ and is not achieved if $\rho \leq \rho_0$. This property will play a key role in the long-time dynamics of solutions to the time dependent problem (1). Moreover, in order to prove the scattering result as introduced in Lecture 1, we will study a more general version of (4) with an energy functional with arbitrary coefficients.

Lecture 3: L^2-scattering for small mass.

Abstract: The last lecture will be devoted to the proof of the property (3) for initial data in Σ_x with sufficiently small L^2-norm. For certain positive parameters $A, \delta(q), \delta(p)$, we consider the auxiliary energy-type functional

$$
\begin{aligned}
E_A(\tau, \varphi(\tau)) = {} & \frac{(1-\tau)^A}{2}\|\nabla_\xi \varphi(\tau)\|_2^2 + \frac{(1-\tau)^{A-\delta(q)}}{q+1}\|\varphi(\tau)\|_{q+1}^{q+1} \\
& - \frac{(1-\tau)^{A-\delta(p)}}{p+1}\|\varphi(\tau)\|_{p+1}^{p+1}.
\end{aligned} \tag{5}
$$

By means of a suitable integral equation jointly with the variational analysis of Lecture 2, we will prove controls of the form

$$
\begin{aligned}
\|\nabla_\xi \varphi(\tau)\|_2^2 &\lesssim (1-\tau)^{-A}, \\
\|\varphi(\tau)\|_{q+1}^{q+1} &\lesssim (1-\tau)^{\delta(q)-A}, \\
\|\varphi(\tau)\|_{p+1}^{p+1} &\lesssim (1-\tau)^{\delta(p)-A},
\end{aligned} \tag{6}
$$

for solutions to (2), provided that $\|\varphi_0\|_2 \leq \rho^* = \rho^*(A, d, p, q)$. Estimates in (6) will enable us to show scattering via a Tsutsumi-Yajima argument.

♢ Paolo Facchi ♢

University of Bari, Italy

Which symmetries survive?
Robust conservation laws in quantum dynamics

◆ Minicourse I	July 21, Tuesday	15:00 – 16:30
◆ Minicourse II	July 22, Wednesday	09:45 – 11:15
◆ Minicourse III	July 22, Wednesday	15:00 – 16:30

Abstract: Symmetries and conservation laws are central to quantum mechanics, but exact conservation is often an idealization: real Hamiltonians are affected by perturbations. This minicourse addresses the question: which quantum symmetries survive perturbations, and in what sense?

We begin with the distinction between fragile and robust conserved quantities, showing that the observables robust against arbitrary perturbations are precisely the functions of the Hamiltonian. We then develop the algebraic characterization of robust quantum symmetries for a given perturbation or family of perturbations, using perturbation-induced refinements of spectral projections. Finally, we introduce the wandering range, which measures how far a robust symmetry can deform, and explain its relation to adiabatic invariants and quantum KAM-type stability.

The course will emphasize the interplay between spectral theory, perturbation theory, symmetry algebras, and long-time quantum dynamics.

♢ Saverio Pascazio ♢

University of Bari, Italy

Descending Dimensions: Exploring Lower-Dimensional Physics via Hadamard ’ s Method of Descent

◆ Minicourse I	July 21, Tuesday	10:30 – 12:00
◆ Minicourse II	July 21, Tuesday	16:45 – 18:15
◆ Minicourse III	July 22, Wednesday	13:15 – 14:45

Abstract: We revisit Ehrenfest's question of why physics appears tied to three spatial dimensions, using Hadamard's method of descent to construct lower-dimensional theories from higher-dimensional ones. By assuming uniformity along extra spatial directions and integrating them out, electromagnetism in 3+1 dimensions yields distinct lower-dimensional sectors. The approach is applied to classical electromagnetism, Newtonian gravitation (with finite speed propagation), and Dirac–Maxwell theory, revealing how dimensional reduction can split, couple, or simplify physical sectors.